Consider a function with more than one independent variable.

e.g.

$$y = f(x,z)$$

$$Q_A = f(P_A, P_O, P_B, Y)$$

$$Q_A = 100 - 2P_A + \tfrac{1}{2}(P_o)^{0.5}$$

What if there is more than one independent variable?

Say $y=f(x,z)$. Then we can talk about the slope of the function as x changes holding z fixed, and the slope of the function as z changes holding x fixed.

The partial derivates of y w.r.t. x and y w.r.t z:

$$\frac{\partial y}{\partial x}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x,z)-f(x,z)}{\Delta x}$$

$$\frac{\partial y}{\partial z}=\lim_{\Delta z\to 0}\frac{f(x,z+\Delta z)-f(x,z)}{\Delta z}$$

Example of partial derivatives:

$$y=7x+12z^2-xz$$

So the partial derivative with regards to x (i.e. holding z fixed) is:

$$\frac{\partial y}{\partial x}=7-z$$

The partial derivative with regards to z (i.e. holding x fixed) is:

$$\frac{\partial y}{\partial z}=24z-x$$

Again, these partial derivatives are functions of two variables so we can take second partial derivatives